Review of progress in Quantitative Nondestructive Evaluation Vol. 15
Edited by D.O. Thompson and D.E. Chimenti (Plenum Press, New York, 1996) pp 2165-2171

OPTICAL LOCK-IN VIBRATION DETECTION USING PHOTOREFRACTIVE FOUR-WAVE MIXING

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INTRODUCTION

Recently many important applications for photorefractive crystals (PRCs) have been found by various investigators.[1-3] These applications range from volumetric information storage in optical computing to adaptive, remote detection of ultrasonic vibration in optical nondestructive evaluation. In this paper, we propose the use of PRC's for lock-in detection of continuously vibrating structures.

The method of photorefractive optical lock-in detection was first demonstrated by Jehad Koury et al at Tufts University in 1991.[4] In their paper, they presented a first-step analytical model which described the temporal behavior of photorefractive lock-in devices, along with experimental verification of this behavior. Also, the effects of varying different mixing parameters were given a qualitative description.

In our work, we have attempted carry the concept of optical lock-in methods further by attempting to quantitatively link optical wave mixing in photorefractive media with classical diffraction theory. In addition, we have performed a shot-noise limited signal-to-noise ratio analysis of this device to determine the minimum detectable displacement sensitivity. In so doing we hope to demonstrate its usefulness to vibration detection and, in particular, its potential for narrowband vibration mode spectral analysis.

EXPERIMENTAL SETUP

The wave mixing process for the vibration detection using photorefractive crystals is shown in figure 1. In this setup, the interference of signal beam 1 and reference beam 2 generates a space-charge field which modulates the local refractive index (generates a

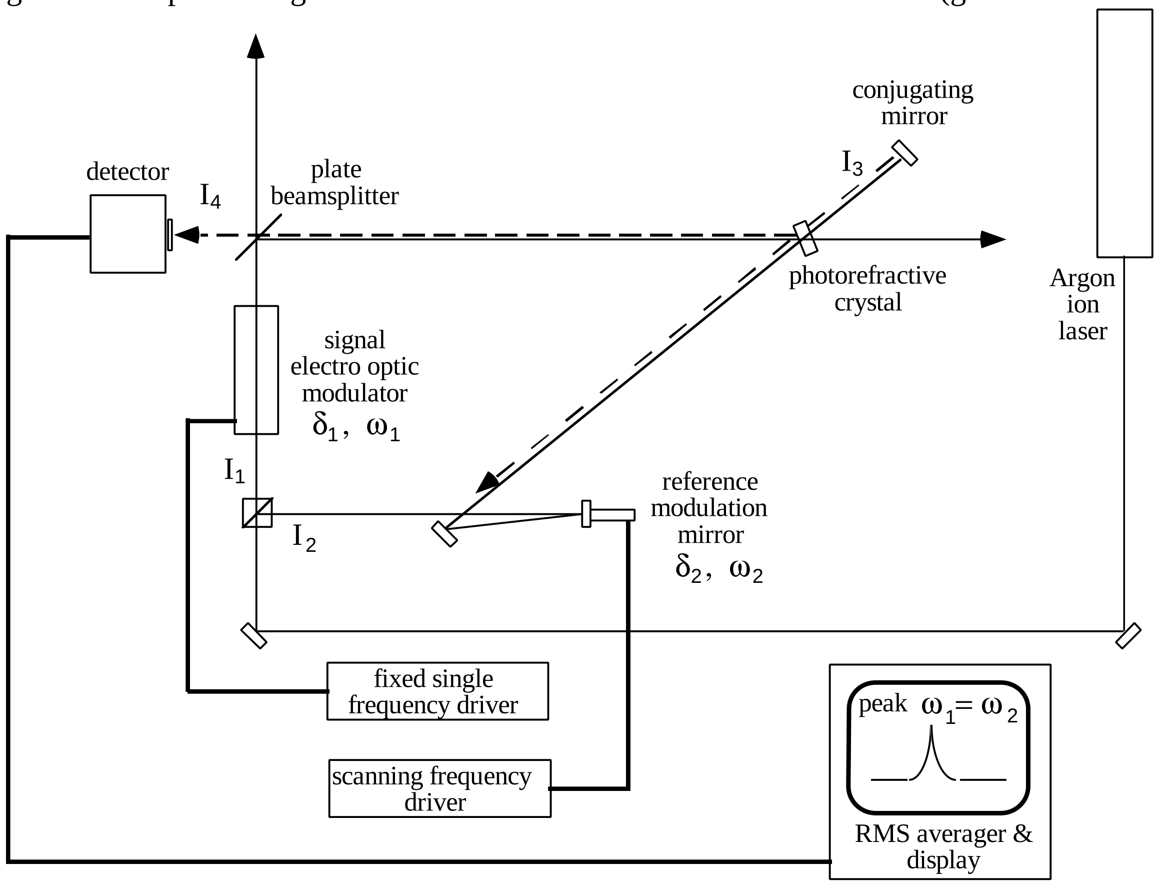

Figure 1 - Setup - optical lock-in detection using photorefractive four-wave mixing.

diffraction grating) within the photorefractive crystal. A counter propagating readout beam 3, antiparallel to reference beam 2 is then sent back into the crystal which upon diffraction by its passage through the grating in the PRC generates a conjugate beam 4. In the following section we show that the amplitude of conjugate beam 4 is dependent on the mixing process and that it is related to the amplitude of the space-charge field. Note that readout beam 3 in the setup passes through a 1/4 wave plate and is reflected from a mirror back into the crystal to insure that readout beam 3 is no longer of the same polarization as beams 1 and 2 such that these two mixing waves form the only grating in the medium, i.e. making the so-called single grating approximation applicable. Under these conditions, the time dependent behavior of the space-charge field is governed by the time response of the material and the instantaneous phase and intensity of the mixing waves.

LOCK-IN MODEL

To begin modeling the lock-in process the assumption that only interference of beams 1 and 2 generates the space-field within the PRC is made. The governing equation used for linking the behavior of the space-charge field E_{sc} and the mixing waves A_1 and A_2 is the following first order partial differential equation

$$\frac{\partial E_{sc}}{\partial t} + \frac{E_{sc}}{\tau} = \frac{E_{sc}^{\max}}{\tau}\frac{\left(A_1 \cdot A_2^{*}\right)}{I_{sat}} \quad \text{where} \quad E_{sc}^{\max} = \frac{q\rho_o L}{2\pi\varepsilon} \quad \text{and} \quad |A_1||A_2| \le I_{sat} \tag{1}$$

where τ is the material response time, t is time and I_{sat} is the light intensity necessary to mobilize all charge carriers within the PRC, saturating the medium. Furthermore, the maximum achieveable space-charge field $E_{sc}^{\max}$ in our model is controlled by the concentration of available charge carriers ρ_o, the fringe spacing L, the carrier charge q and the permittivitiy of the medium ε.[3] The mixing waves, signal and reference, each have their own unique phase modulation parameters and are given by

$$A_1 = |A_1| e^{\left[-i\delta_1 \sin(\omega_1 t + \phi_1)\right]} = |A_1| \sum_{n=-\infty}^{\infty} J_n(\delta_1) e^{\left[-in(\omega_1 t + \phi_1)\right]} \tag{2a}$$

$$A_2 = |A_2| e^{\left[-i\delta_2 \sin(\omega_2 t + \phi_2)\right]} = |A_2| \sum_{m=-\infty}^{\infty} J_m(\delta_2) e^{\left[-im(\omega_2 t + \phi_2)\right]} \tag{2b}$$

where δ_1, ω_1 and δ_2, ω_2 are the amplitudes and frequencies of the phase modulations imposed by the electro-optic modulator (signal) and the piezo-electric mirror (reference), respectively. Additionally, ϕ_1, and ϕ_2 are the phases of the two phase modulated mixing beams. The relative phase in our application is usually meaningless since the frequency of the reference beam is being scanned, however, the detector is phase-sensititve when the frequency difference is zero.

In order to find the relation between space-charge field and the mixing waves we insert (2a) and (2b) into (1) and use an integrating factor. Making the same assumption as Koury et al about the low-pass filtering behavior of the PRC which is only capable of responding to the first few frequency difference components of the optical interference occuring within the response time of the material, the space-charge field becomes[4]

$$E_{sc}(t) = E_{sc}^{\max}\frac{|A_1||A_2|}{\tau\, I_{sat}} \sum_{n=-\infty}^{\infty} \frac{J_n(\delta_1) J_n(\delta_2)}{in\Delta\omega + 1/\tau}\, e^{(-in\Delta\omega t)}\, e^{(-in\Delta\phi)} \tag{3}$$

where $\Delta\omega$ is the frequency difference, $\Delta\phi$ is the phase difference and J is the Bessel function of the first kind.

The space-charge field can then be related to modulation of the local refractive index through the linear electro-optic effect.[5] This relation gives rise to an important parameter in diffractive optics known as the wave coupling constant. In our case, however, this quantity is a function of time such that we call it the 'wave coupling parameter' and define it as

$$\nu = \frac{\pi N^3 r_{41} L}{2 \lambda \cos\theta} E_{sc}(t) \tag{4}$$

where N is average refractive index of the medium, r_{41} is the effective, orientation-dependent electro-optic coefficient in an anisotropic crystal, λ is the source wavelength and θ is angle between the mixing waves. Wave coupling in this lock-in model occurs between the readout beam 3 and the diffracted (generated) beam 4, while the signal and reference beams act simply to generate the spatially modulated refractive index (coupling effects between these waves are ignored). The intensity of beam 4 is related to the coupling parameter and has been well described by Kogelnik[6]

$$I_4 = I_3 \, e^{(-2\alpha L/\cos\theta)} \sin^2 \nu \tag{5}$$

where α is the material absorption. Substituting the solution for the space-charge field (3) into the wave coupling parameter (4) and exploring only the summation terms of n equal to -1, 0 and 1 yields a solution for the intensity of beam 4 given by

$$I_4 = C_o I_3 \left\{ \frac{C_1^2 L^2}{\tau^2} \left[\tau^2 J_0^2(\delta_2) + \frac{2\tau \delta_{min} J_0(\delta_2) J_1(\delta_2)}{1/\tau^2 + \Delta\omega^2} F + \frac{\delta_{min}^2 J_1^2(\delta_2)}{\left(1/\tau^2 + \Delta\omega^2\right)} F^2 \right] \right\} \tag{6}$$

where $F = 1/\tau \cos(\Delta\omega t + \Delta\phi) - \Delta\omega \sin(\Delta\omega t + \Delta\phi)$ (7a)

$$C_0 = e^{(-2\alpha L/\cos\theta)} \quad \text{and} \quad C_1 = \frac{N^3 r_{41} q r_o L |A_1||A_2|}{4 \lambda \varepsilon \cos\theta I_{sat}} . \tag{7b \& c}$$

where we have only used the first term in the series representation of the Bessel functions with δ_1 arguments, and renamed δ_1 to δ_{min} for reasons that will become apparent later.

Inspection of expression (6) shows that the intensity of beam 4 is made up of a DC offset intensity term, an AC beat intensity term and a higher order term. In the following section, we explore the effects of phase modulation amplitude on the DC and AC intensity components of the photorefractive lock-in detection technique, and disregard the higher order term. Furthermore, we use a shot-noise limited criteria to define the signal-to-noise ratio and minimum detectable displacement for this method.

SIGNAL-TO-NOISE RATIO

In this development of the signal-to-ratio we use the same method as Wagner.[7] This involves the ratio of the peak-to-peak signal intensity squared over the operating point intensity. In our case, the operating point intensity or offset intensity in this optical lock-in device is given by

$$\overline{I}_4 = C_o \, C_1^2 \, I_3 \, L^2 \, J_0^2(d_2) \tag{8}$$

or the first term in expression (6). The signal peak-to-peak intensity or beat intensity is given by

$$DI_4 = 2 \, d_{min} C_o \, I_3 \, C_1^2 \, L^2 \, \frac{J_0(d_2) J_1(d_2)}{1 + Dw^2 t^2} \, t \, F \tag{9}$$

For the application of vibration mode spectral analysis we are interested in the RMS beat intensity value of (9) given by

$$DI_4 = 2 \, d_{min} C_o \, I_3 \, C_1^2 \, L^2 \, \frac{\sqrt{2}/2 \, J_0(d_2) J_1(d_2)}{\sqrt{1 + Dw^2 t^2}} \tag{10}$$

The definition of the signal-to-noise ratio, as described earlier, in the shot-noise limited detection case is given by

$$SNR = C_2 \, (DI_4)^2 / \overline{I}_4 \quad \text{where} \quad C_2 = \frac{h}{2 \, h n_p \, B} \tag{11a \& b}$$

and where h is the efficient of the detector, hn_r is the photon energy of the source and B is the bandwidth of the crystal. Substituting (8) and (10) into expression (11) yields a shot-noise limited signal-to-noise ratio given by

$$SNR = 4 \, d_{min}^2 C_o \, C_1^2 \, C_2 \, I_3 \, L^2 \left(\frac{\sqrt{2}/2 \, J_1(d_2)}{\sqrt{1 + Dw^2 t^2}} \right)^2 \tag{12}$$

By setting the signal-to-noise ratio equal to one we can determine the minimum detectable displacement d_{min} given by[7]

$$d_{min} = \frac{1}{\sqrt{2} \, C_1 L} \, \frac{\sqrt{1 + Dw^2 t^2}}{J_1(d_2)} \, \sqrt{\frac{1}{C_o \, C_2 \, I_3}} \tag{13}$$

As an example of the use of this expression we define some realistic parameters (listed below) for the application of this device (operating in saturation) and calculate d_{min}.

t = 0.01 (s)	q = 1.609*10^{-19} (C)
L = 10^{-6} (m)	r_o = 10^{22} (m^{-3})
Dw = (radians•s^{-1})	e = 56•e_o = 56*8.85*10^{-12} (C^2•N^{-1}•m^{-2})
q = π/6 (radians)	N = 2.54 (unitless)
r_{41} = 5*10-12 (m•V^{-1})	l = 514*10^{-9} (m)
a = 0.0005 (m^{-1})	F = π/2 (radians)
I_3 = 10^6 (Watts•m^{-2})	L = 5*10^{-3} (m)
hn_r = 3.83*10^{-11} (Joules) (514 nm)	h = 10% (unitless)

The calculated minimum detectable displacement using this model is approximately $1^{x}10^{-5}$ μm per Hz$^{1/2}$. Preliminary work in the laboratory has demonstrated subnanometer displacement sensitivity for continuous vibration measurement. This experimental result was obtained using an electro-optic device to generate a known, quantitative phase excursion in the signal beam while the system was operating in an optimum configuration. This optimum operation will be explained in the next section.

EXPERIMENTAL RESULTS - MODEL VERIFICATION

In this section, the effects of phase modulation amplitude in the photorefractive optical lock-in device are investigated experimentally and compared to analytical results. In the experimental procedure the phase modulation amplitude of the signal beam was held constant while the amplitude of the reference beam was incremented. Also, the frequency difference between the two mixing waves was held constant at 50 Hz, while the carrier frequency was held at 5 KHz, i.e. the signal beam was phase modulated at 5 KHz and the reference beam was phase modulated at 5.05 KHz. The mixing was done in a Bizmuth Silicon Oxide PRC with an inverse response time of approximately 90 Hz.

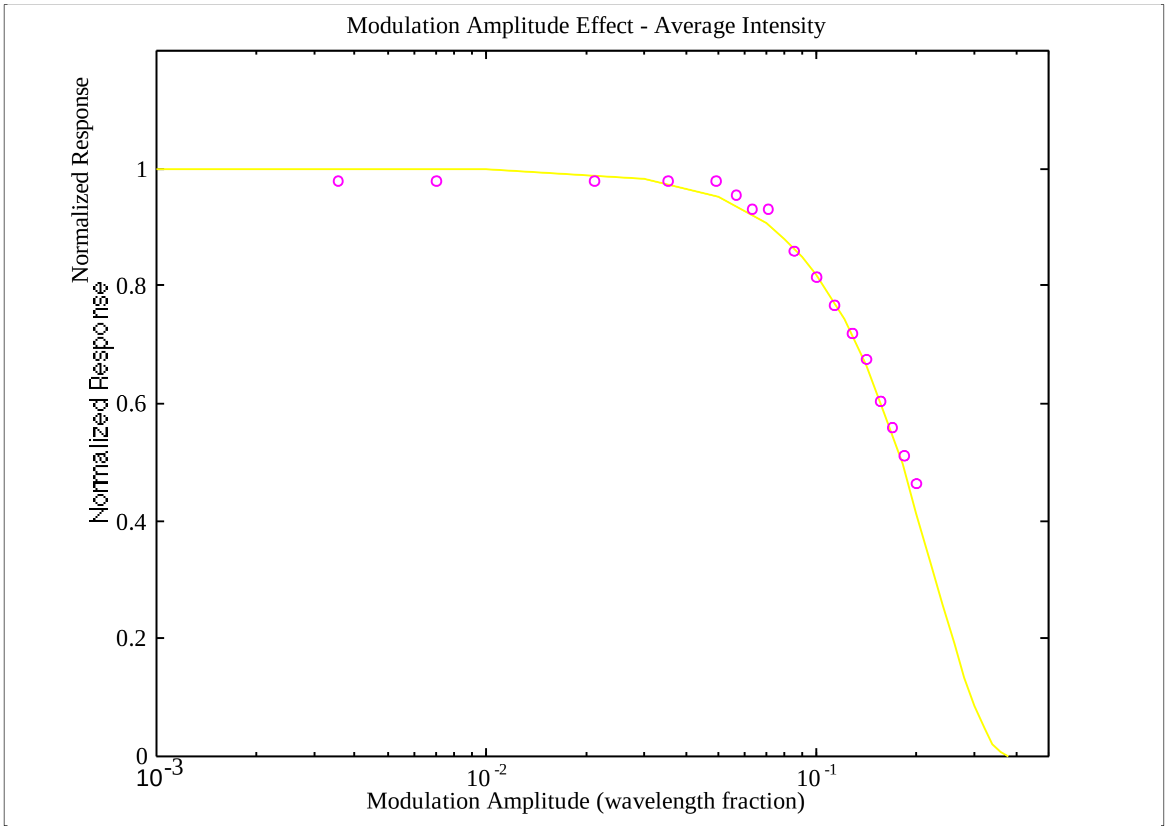

Figure 2 - Experimental verification of the DC offset intensity behavior.

In figure 2, the DC offset intensity was obtained by putting the output of the detector into a low-pass filter set with a corner frequency of approximately 10 Hz. The reference beam phase modulation amplitude was varied by fractions of an optical wavelength using a calibrated piezo-electric translation mirror. The results of this experiment have been normalized and show that amplitude behavior of the device is well described by the analytical model. From the figure it can be seen that large phase incursions around 1/10th of a wave begin to drive the DC offset intensity down.

In figure 3, the AC beat intensity was obtained by putting the output of the detector into a high-pass filter set with a corner frequency of approximately 30 Hz. Again, the reference beam phase modulation amplitude was incrementally varied while the peak-to-peak signal intensity was obtained. The results of this experiment have been normalized, and show that amplitude behavior of the device is again well described by the analytical model. The peak in the experimental data in the figure shows that there is an optimum modulation depth for the reference beam at approximately 2/10ths of a wave. This observation is also brought out in the AC component of the analytical model (expression (9)) where the product of the two Bessel functions have the same peak. The significance of this observation is that a photorefractive optical lock-in vibration detector has an optimum operating configuration.

VIBRATION MODE SPECTRAL ANALYSIS

After experimentally verifying the behavior of the lock-in device using the experimental equipment available, we further explored its potential applications. The

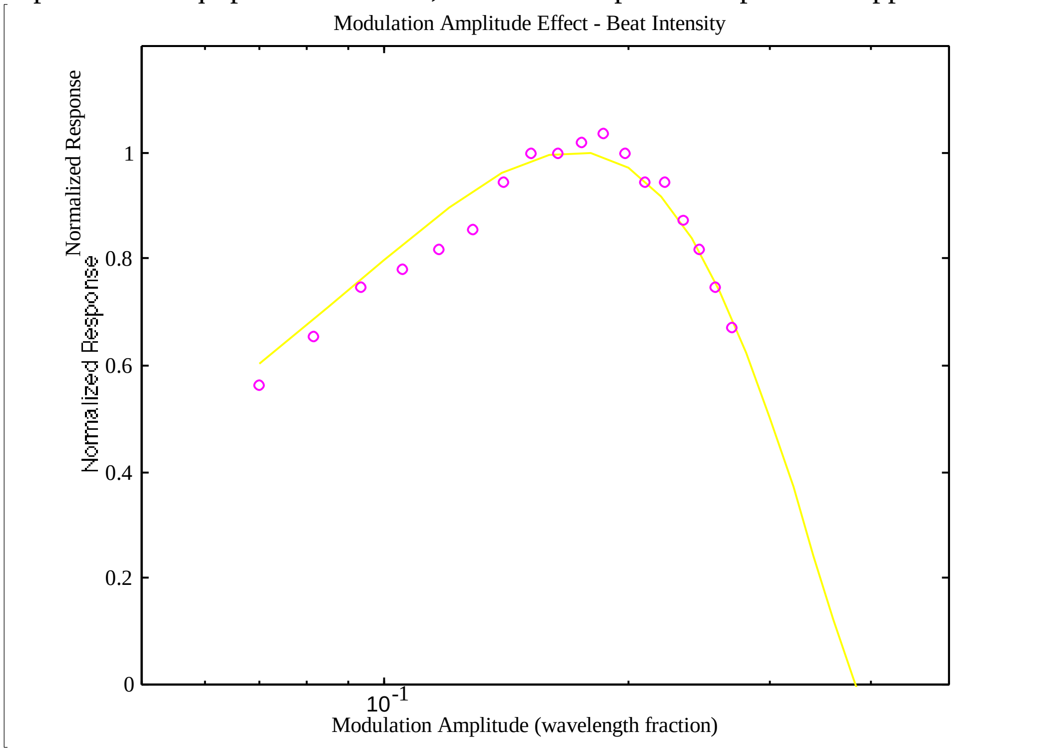

Figure 3 - Experimental verification of the AC beat intensity behavior.

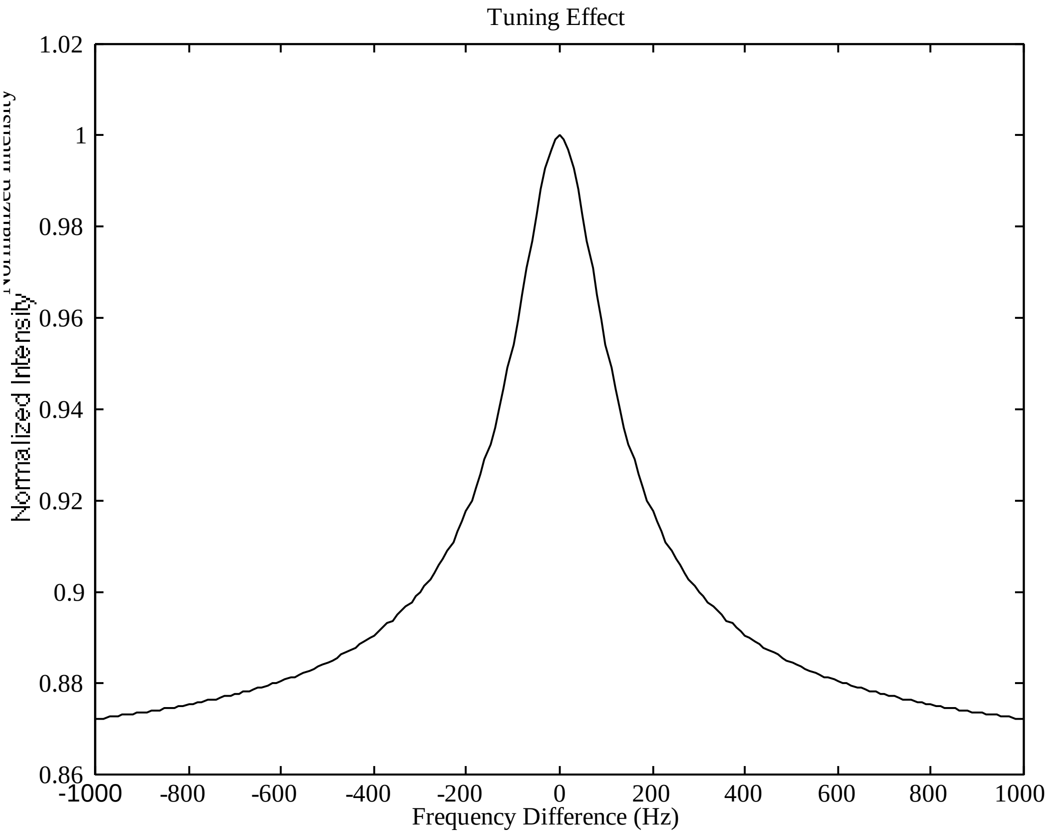

Figure 4 - Simulated vibration mode spectrum, i.e, tuning effect

frequency matching character of the device demonstrated in earlier experiments suggests that it could be used as a vibration mode spectrum analyzer. To further investigate this prospect we used the analytical model to describe a tuning effect present in expression (6) as a function of frequency difference between the mixing waves. This effect is shown in figure 4 for a system using a PRC with a time response of 0.01 seconds. The peak in this tuning curve occurs at *Dw* equal to zero and has a full width at half maximum of approximately 100 Hz. The significance of this result is that this device could be used as vibration spectrum analyzer with mode discrimination based on the bandwidth of the crystal used. However, device design issues such as frequency scanning rate become important since a crystal with high discrimination like $BaTiO_3$ (*B* equal to 1.0 - 0.1 Hz) require a relatively slow scan before an adequate response can be detected. In the future, we are planning to generate this spectral response curve using a RMS voltage monitoring device coupled to the experiment.

CONCLUSIONS

The phase modulation amplitude effects presented in the analytical lock-in model agree very well with the experimental results obtained. The optical lock-in vibration displacement sensitivity of the device has been demonstrated to be in the subnanometer range, and the theoretical analysis suggests that the device is capable of providing mucher higher sensitivity. Potential applications of a photorefractive optical lock-in device may be in all optical narrowband vibration mode spectral analysis.

ACKNOWLEDGEMENTS

Work supported through the INEL Laboratory Directed Research & Development Program under DOE Idaho Operations Office Contract DE-AC07-94ID13223.

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